University of Waterloo, Winter 2021
AMATH741/CM750/CS778: Numerical Solution of Partial Differential Equations

Instructor: Prof. Hans De Sterck, **email:** hdesterck@uwaterloo.ca
Lecture materials: will be delivered online (pdf board notes and course notes)
Live-streamed flipped-classroom sessions (Zoom, optional attendance, recorded): Wednesdays and Fridays 10:00-11:20am (subject to change)
TA: Esha Saha, **email:** esaha@uwaterloo.ca
Instructor Office hours: Wednesdays 9:00-9:45am, Thursdays 11:00am-12:00 (Zoom, time to be confirmed)
TA Office hours: Tuesdays 10:00am-11:00, Thursdays 9:00-10:00am (MS Teams, time to be confirmed)

Course description and objectives:
Many problems in science, engineering and finance can be formulated in terms of partial differential equations (PDEs). Since analytical solutions are not available in general, it is necessary to use numerical methods to approximate the solution. This course will cover the basic techniques for solving PDEs numerically.
The goal of the course is threefold. You will receive a solid introduction to the theory of numerical methods for partial differential equations (with derivations of the methods and some proofs). You will learn to implement the computational methods efficiently in Matlab or another language of your choice, and you will apply the methods to problems in several fields, for example, fluid mechanics, diffusion processes and wave phenomena.
This is a graduate-level numerical methods course that will be useful for students from Applied Mathematics, Computational Mathematics, Computer Science, Data Science, Quantitative Finance, and other programs in Mathematics, Science and Engineering.

Prerequisites: Some previous experience with numerical computation and programming, and some background in PDEs, is desirable. (Also: it is not recommended to take this course if you have previously taken a numerical PDEs course, such as AMATH442 at University of Waterloo.)

Tentative outline:
1. Finite Difference (FD) Methods (4 weeks)
2. Finite Volume (FV) Methods (4 weeks)
3. Finite Element (FE) Methods (4 weeks)

References:
- Numerical Mathematics, Quarteroni, Sacco and Saleri, Springer, 2007
  (You can download pdf files for this book on campus at link.springer.com.
  For off-campus access, you first have to login via the library website, see
  https://login.proxy.lib.uwaterloo.ca/login
  which will allow you to access the book's proxy URL
  https://link-springer-com.proxy.lib.uwaterloo.ca/book/10.1007/b98885 .)

- Numerical partial differential equations: Finite Difference Methods, J.W. Thomas, Springer, 2004. (FD) (also available from link.springer.com)
- The mathematical theory of finite element methods, Brenner and Scott, Springer, 1994. (FE, theoretical) (also available from link.springer.com)
- A first course in the numerical analysis of differential equations, Iserles, Cambridge University Press, 1997. (FD and FE, Chapters 7-14)
- Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics, by Dietrich Braess, Cambridge University Press, 2001. (FE, theoretical)
- An introduction to the finite element method, Reddy, McGraw-Hill, 1993. (FE, comprehensive introduction with engineering applications)
- Finite volume methods for hyperbolic problems, Leveque, Cambridge, 2002. (FV)
- Numerical Methods for Conservation Laws, Hesthaven, SIAM, 2018.

Course Website: the LEARN system will be used extensively for all course communications. Zoom and/or MS Teams will be used for life-streamed course components and office hours. Crowdmark will be used for administering written assessments.

How the class will be taught:

- The class is fully online.
- The primary mode of delivery of the course materials is a combination of **hand-written "board notes" posted on LEARN** and **live-streamed "flipped-classroom sessions"** (these Zoom sessions will be recorded with the recordings posted on LEARN, and attendance is fully optional).
- The **handwritten "board notes"** are the notes the instructor would have written on the board in an on-campus offering, and contain all the primary course material. *You will be asked to read and study the relevant section of the board notes before the corresponding flipped-classroom session.* In the **flipped-classroom session** the instructor will go over the board notes highlighting the big lines and the important or tricky parts (at a faster speed than a regular board lecture), and students will have the opportunity to ask live questions (via video, voice or text chat) about the board notes and about other aspects of the course (like assignment questions etc.). These sessions are scheduled for a length of up to 80min, but it is expected they will often take substantially less time, since you will have studied the material beforehand, and some sessions will treat topics that are shorter than other sessions.
- **Typed course notes** will also be posted on LEARN, to complement the hand-written board notes and flipped-classroom sessions.
- **Note:** to enhance social interaction with your classmates, we will start each flipped-classroom session 15 minutes early (from 9:45am) for a **15-minute social chat** during which you will be encouraged to switch on your camera and microphone. There will also be an opportunity for further questions or social discussion at the end of each flipped-classroom session.

Course components – final mark breakdown:

3 theoretical assignments (individual) of equal weight	25%
3 computational assignments (individual) of equal weight	25%
group project (groups of 3)	25%
take-home final test (during final exam period)	25%

Assignments:

- There will be **three individual theoretical assignments** of equal value (25% weight), administered through Crowdmark.
- There will be **three individual computational assignments** of equal value (25% weight), administered through LEARN and Crowdmark.
- You can take a three calendar-day extension on one theoretical assignment and on one computational assignment (no questions asked), but you have to notify the instructor of this ahead of the assignment due date.
- Matlab is the recommended computer language for the computational assignments, but you can choose to use a different language like python or Julia or C or C++ (contact the instructor if you want to choose a language not listed; Maple or Mathematica are not suitable). Instructions about installing Matlab on your computer and accessing the Waterloo campus licence, or running Matlab remotely, will be given in Assignment 0 (not to be handed in).
- Matlab is recommended because some of the questions may be written in part assuming you use Matlab, and for some questions some short Matlab code fragments may be provided on LEARN for download. So if you do not have much programming experience, Matlab is recommended. But there should not be an issue if you prefer to use another language that you are experienced in: it should not be difficult to translate the few code snippets that will be provided in Matlab to another language like python, and you can consult with other students (e.g., on piazza) about translating the Matlab that is provided to python or another language.

Group project: (25% weight, groups of 3)

- Each group will choose a research paper on an application area and a numerical PDE method that is of interest to the group members (e.g., in your area of research).
- The project goal is to gain understanding of the contents of the paper (understand the PDE problem and the numerical method) and to try out the numerical method computationally, reproducing some of the results of the paper and trying out at least one aspect that extends the paper (e.g., different problem parameters, different grid size, different order of accuracy, etc.).
- Groups are to be formed by Wednesday February 3.
- An 1-page project proposal (identifying the research paper and introducing the application problem/PDE and the numerical PDE method that is used) is due on Wednesday February 24 (after the reading week).

- Groups present their results in 12-minute presentations (each student speaks approximately 4 minutes) on Friday April 9 and Wednesday April 14 (during the usual lecture times). (Weight 10%.)
- Groups submit a project report, in research paper format, by Wednesday April 14. The report (10 pages) contains, on page 1, the paper title, author names, and paper abstract, and a high-level introduction section (on the PDE problem and the numerical method used). This is followed by a 4-page technical description of the PDE problem and the numerical method, 4 pages of numerical results with discussion (including figures and tables), and 1 page for conclusion and references. (Weight 15%.)

Final test: Written take-home test (25% weight).

- This will be an open-book test administered on Crowdmark. You can consult all class materials posted on LEARN and your own notes for the course (including posted board notes, pdf course notes, your own notes, assignment solutions posted on LEARN, video recordings posted on LEARN). You will not be allowed to consult online or off-line sources beyond the LEARN course materials, and interaction with others about the final test is not allowed.
- You can choose the start time of your test within a 2-day window, and you will have 24 hours to complete your test once started. The final test will not contain programming questions.

Assessment schedule (NOTE: DATES ARE TENTATIVE AND SUBJECT TO CHANGE)		
semester week	**date**	**assessments due**
week 3	Fri Jan 29	assignment C1 (computational)
week 4	Wed Feb 3	project groups formed (groups of 3)
week 5	Fri Feb 12	assignment T1 (theoretical)
READING WEEK	February 15-19	(no classes)
week 6	Wed Feb 24	1-page project proposal due
	Fri Feb 26	assignment T2 (theoretical)
week 8	Fri Mar 12	assignment C2 (computational)
week 10	Fri Mar 26	assignment T3 (theoretical)
GOOD FRIDAY	Fri Apr 2	(no class)
week 12	Wed Apr 7	assignment C3 (computational)
	Fri Apr 9	project presentations (no lecture)
week 13	Wed Apr 14	project presentations (no lecture) project report due
exam period	Wed-Fri Apr 21-23	take-home test

Office hours and student forum: Regularly scheduled **individual office hours** will be offered online via Zoom or MS Teams (using video, voice or chat). There will also be an active **piazza forum** where students, TA and instructor provide answers to student questions.

Recording of online course components:

- The **regularly scheduled flipped-classroom sessions** (optional attendance) will be **recorded and posted on LEARN**. Student questions will only be recorded when the student chooses to use video and/or voice when asking a question (questions via chat will not be recorded). During the flipped-classroom sessions you will normally mute your camera and microphone (except, if you wish, while asking a question).
- Of course, office hours will not be recorded.

Late Assignments/Missed Exams: Beyond the above-mentioned extension on one theoretical and one computational assignment, there will be no extensions for assignments except under extenuating (and documented) circumstances. If you are ill, please be prepared to provide a note from the health center or your doctor, see https://uwaterloo.ca/math/accommodations (but if you are affected by Covid-19, you can self-declare.). If your instructor decides that your circumstances warrant special accommodation, your final grade will be calculated based on your performance on the remaining assignments. When possible, advance notice must be given.
If you miss the final test (due to illness, or other documented and approved extenuating circumstances), the weight of the test will be shifted to an oral exam.

Grade Appeals: We will make every effort to be fair and consistent in the marking. Grade appeals must be submitted to the TA via email within one week from the date the assignment was returned. For this reason, it is important that you look over all returned assignments on the day they are returned.

Academic Integrity Policy for Assignments: All submitted assignment solutions should be strictly your own work. For the theoretical and computational assignments, you are allowed to discuss the assignment problems with your classmates at a general level (but not step-by-step). You are not allowed to copy any material. You are not allowed to show parts of your written assignment to another student. For programming questions, you are allowed to discuss issues you encounter with others, but you are not allowed to show others your computer code or copy computer code from others. You are not allowed to copy theoretical solutions or computer code from online resources. All assignment material you submit (including written documents, program code and graphical output) should be strictly your own work. Compliance will be actively monitored. Instances of suspected cheating will be dealt with seriously, in accordance with Faculty and University policies.

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check the Office of Academic Integrity for more information.]

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4. When in doubt,

please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for his/her actions. [Check the Office of Academic Integrity for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline. For typical penalties, check Guidelines for the Assessment of Penalties.

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals.

Note for students with disabilities: AccessAbility Services, located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

Mental Health: If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support.

UWaterloo Resources

- Campus Wellness: *https://uwaterloo.ca/campus-wellness/students*
- Counselling Services: counselling.services@uwaterloo.ca, *519-888-4567 ext 32655*
- Health Services: 519-888-4096.

Off-Campus Resources

- Good2Talk (24/7): Free confidential help line for post-secondary students. Phone: 1-866-925-5454, https://good2talk.ca/
- Here 24/7: Mental Health and Crisis Service Team. Phone: 1-844-437-3247 or 519-821-3582, https://here247.ca/
- OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo. Phone: 519-884-0000, https://ok2bme.ca/

Diversity: It is our intent that students from all diverse backgrounds and perspectives be well served by this course, and that students' learning needs be addressed both in and out of class. We recognize the immense value of the diversity in identities, perspectives, and contributions that students bring, and the benefit it has on our educational environment. Your suggestions are encouraged and appreciated. Please let us know ways to improve

the effectiveness of the course for you personally or for other students or student groups. In particular:

- We will gladly honour your request to address you by an alternate/preferred name or gender pronoun. Please advise us of this preference early in the semester so we may make appropriate changes to our records.
- We will honour your religious holidays and celebrations. Please inform us of these at the start of the course.
- We will follow AccessAbility Services guidelines and protocols on how to best support students with different learning needs.

Remote Teaching and Learning: STUDENT NOTICE OF RECORDING

Activities for this course involve recording, in partial fulfillment of the course learning outcomes. You will receive notification of recording via at least one of the following mechanisms: within the Learning Management System (LEARN), a message from your course instructor, course syllabus/website, or other means. Some technologies may also provide a recording indicator. Video/audio recorded during flipped-classroom sessions will be made available to students of AMATH741/CM750/CS778 for the purpose of helping in the study of the course materials. Project presentations will not be recorded. If an oral exam is needed, video/audio of the exam will be recorded for record-keeping, unless the student declines recording. Recordings will be managed according to the University records classification scheme, WatClass, and will be securely destroyed when no longer needed by the University. Your personal information is protected in accordance with the Freedom of Information and Protection of Privacy Act, as well as University policies and guidelines and may be subject to disclosure where required by law.

The University will use reasonable means to protect the security and confidentiality of the recorded information, but cannot provide a guarantee of such due to factors beyond the University's control, such as recordings being forwarded, copied, intercepted, circulated, disclosed, or stored without the University's knowledge or permission or the introduction of malware into computer system which could potentially damage or disrupt the computer, networks, and security settings. The University is not responsible for connectivity/technical difficulties or loss of data associated with your hardware, software or Internet connection.

By engaging in course activities that involve recording, you are consenting to the use of your appearance, image, text/chat messaging, and voice and/or likeness in the manner and under the conditions specified herein. (In the case of a live stream event, if you choose not to have your image or audio recorded, you may disable the audio and video functionality (see: *Student privacy during live events*). Instructions to participate using a pseudonym instead of your real name are included where the feature exists; however, you must disclose the pseudonym to your instructor in advance in order to facilitate class participation.) This notice serves as confirmation that you can choose to participate in any course component without being recorded.

You are not permitted to disclose the link to/URL of an event or an event session recording or copies of recording to anyone, for any reason. Recordings are available only to authorized individuals who have been directly provided the above instructions/link for their use. Recordings for personal use, required to facilitate your learning and preparation of personal course/lecture notes, should not be shared with others without the permission of the instructor or event coordinator. Review the University's guidelines for faculty, staff and students entering relationships with external organizations offering access to course materials for more information on your obligations with respect to keeping copies of course materials. For more information about accessibility, connect with AccessAbility Services.